**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

from scipy.stats import norm

m = 55 # mean 45 minutes + 10 min(10 minutes after the car is dropped off)

std = 8 # standard deviation 8 minutes

x = 60 # car will be ready in 1 hour i.e, 60 minutes

z\_score = (x - m)/std

prob = 1 - norm.cdf(z\_score)

print(f"The probability that the service manager cannot meet the commitment is approximately ",prob.round(3))

**output :** The probability that the service manager cannot meet the commitment is approximately 0.266

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

mean = 38

std = 6

#probability of age less than 38

prob\_less\_38 = norm.cdf(38, loc = mean, scale = std)

# probability of age less than 44

prob\_less\_44 = norm.cdf(44, loc = mean, scale = std)

# probability between 38 and 44

prob\_between\_38\_and\_44 = prob\_less\_44 - prob\_less\_38

#probability of age greater than 44

prob\_great\_44 = 1-norm.cdf(44, loc = mean, scale = std)

print('The probability of employees whose age is more than 44 :-', prob\_great\_44.round(3), '\nThe probability of employees whose age is between 38 and 44 :-',prob\_between\_38\_and\_44.round(3) )

if prob\_great\_44 > prob\_between\_38\_and\_44:

  print('Since, probabilty of age above 44 is greater than the probabilty of age between 38 and 44\nThe given statement is ',True)

else :

  print('Since, probabilty of age above 44 is less than the probabilty of age between 38 and 44\nThe given statement is ',False)

**Output :**

The probability of employees whose age is more than 44 :- 0.159

The probability of employees whose age is between 38 and 44 :- 0.341

Since, probabilty of age above 44 is less than the probabilty of age between 38 and 44 ,The given statement is False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Answer:**

total = 400 # total no.of employees

b= norm.cdf(30, loc = mean, scale = std) # probabbility of employess age less than 30

x = b\*total

print('A training program for employees under the age of 30 at the center would be expected to attract about',x.round(3),'employees')

**Output :**

A training program for employees under the age of 30 at the center would be expected to attract about 36.484 employees

Therefore, the statement is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer:**

**The Normal Distribution is linked to the Central Limit Theorem, which suggests that the sum of many independent, identically distributed random variables approaches a Normal distribution. This applies to (X1 + X2) and (2X1) when X1 and X2 are independent and identically distributed, and the sample size is sufficiently large. The distinction between 2X1 and (X1 + X2) lies in the magnitude they represent for two distinct sample subsets, drawn from the same source. If X1 follows a normal distribution with mean μ and variance σ^2, then 2X1 follows a Normal distribution with parameters (2μ) and (4σ^2), and (X1 + X2) follows a Normal distribution with parameters (2μ) and (2σ^2). Subtraction results in a distribution with parameters (0) and (6σ^2), consistent for every sample subset. The Normal distribution is defined by two parameters: mean (μ) and variance (σ^2).**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer:**

x = norm.interval(0.99, loc = 100, scale = 20)

x = (x[0].round(1), x[1].round(1))

x

Output : (48.5, 151.5)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer:

# For profit1

m1 = 5 \* 45 # convert rupees

std1 = 3

prob1 = norm.interval(0.95, loc = m1, scale = std1)

prob1 = (prob1[0].round(0),prob1[1].round(0))

# For profit2

m2 = 7 \* 45 # convert rupees

std2 = 4

prob2 = norm.interval(0.95, loc = m2, scale = std2)

prob2 = (prob2[0].round(0),prob2[1].round(0))

print('probability for the annual profit of the company with profit1 range', prob1, '\n probability for the annual profit of the company with profit 2 range', prob2)

Output :

probability for the annual profit of the company with profit1 range (219.0, 231.0)

probability for the annual profit of the company with profit 2 range (307.0.0, 323.0)

1. Specify the 5th percentile of profit (in Rupees) for the company

Answer:

ppf1 = norm.ppf(0.05, loc= m1, scale = std1)

ppf1 = ppf1.round(2)

ppf2 = norm.ppf(0.05, loc= m2, scale = std2)

ppf2 = ppf2.round(2)

print('the 5th percentile of profit1 :',ppf1,'\n the 5th percentile of profit2 :',ppf2)

Output :

the 5th percentile of profit1: 220.07

the 5th percentile of profit2: 308.42

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

z\_score\_loss\_profit1 = (0 - m1) / std1

z\_score\_loss\_profit2 = (0 - m2) / std2

prob\_loss\_profit1 = norm.cdf(z\_score\_loss\_profit1)

prob\_loss\_profit2 = norm.cdf(z\_score\_loss\_profit2)

print('Company 1 loss probability: ',prob\_loss\_profit1,'\nCompany 2 loss probability: ',prob\_loss\_profit2 )

Output :

Company 1 loss probability: 0.0

Company 2 loss probability: 0.0